

Components of Vectors

Vectors that are perpendicular to each other are independent of each other, and their effects can be considered separately.

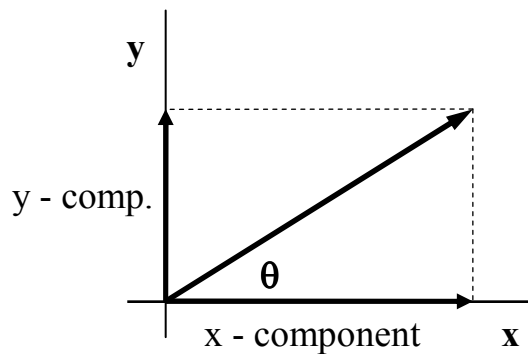
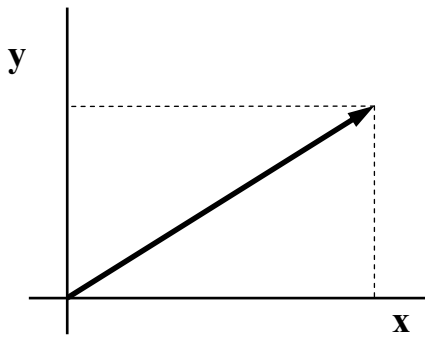
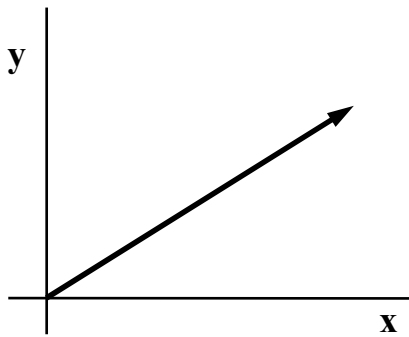
If a vector can be taken apart so that we have only its effects on the X and Y axes, then one-dimensional equations can be used on each axis.

Taking apart a vector like this is called resolving it into perpendicular components. In general, such components are either:

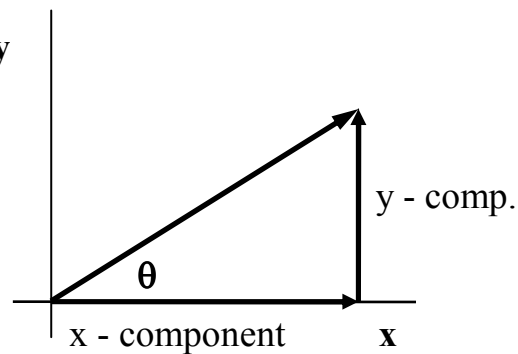
- a) horizontal and vertical, or
- b) parallel and perpendicular to an inclined surface.

To resolve a vector into perpendicular components:

1. Place the vector on an X-Y grid.
2. Drop perpendicular lines to each axis.
3. Draw your components on each axis at the perp. cuts **or** draw the components tip-to-tail as though the vector was a resultant.

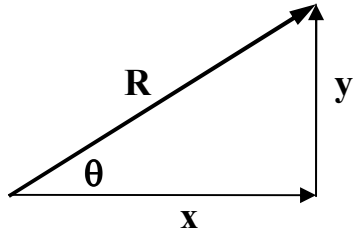


OR



Once a vector is resolved into its components this way, trigonometry can be used to find the magnitude of the components.

Consider the following diagram:



Vector **R** can represent any vector quantity (e.g. force **F**, velocity **v**, etc.). Therefore, **x** and **y** represent the components of the vector quantity.

F --> **F_x**, **F_y** components

v --> **v_x**, **v_y** components

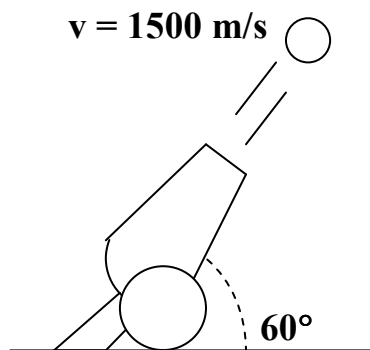
In the diagram above,

$$\sin \theta = \frac{y}{R} \quad \text{therefore} \quad y = R \sin \theta$$

$$\cos \theta = \frac{x}{R} \quad \text{therefore} \quad x = R \cos \theta$$

Example 7.

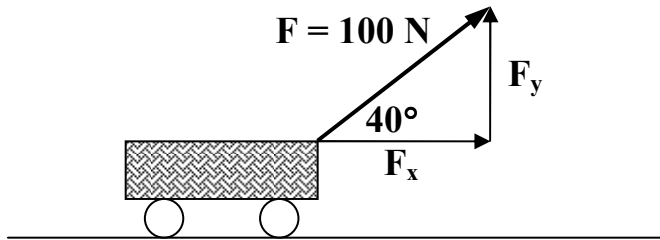
A cannon is shot at a muzzle velocity of 1500m/s at an angle of 60° to the horizontal. What are the vertical and horizontal components of the velocity?



(see Vectors Ex 7 for answer)

Example 8.

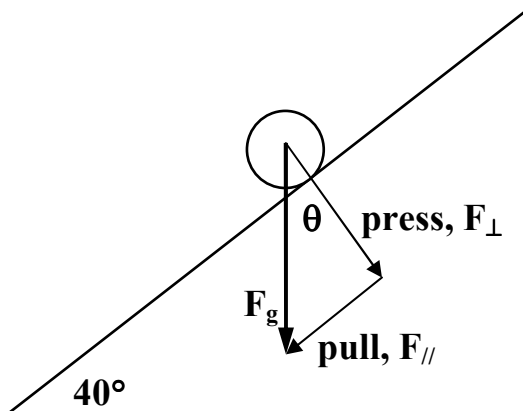
A boy pulls a wagon with a force of 100 N at 40 degrees to the horizontal. Find the pulling force (F_x) and the lifting force (F_y). (64 N, 77 N)



(see Vectors Ex 8 for answer)

Note that components are not always drawn as horizontal or vertical components. Sometimes, it's easier to show components that are *parallel* and *perpendicular* to surface where the object is perched.

Think about this problem: a ball rests on a slope. What component of its weight F_g presses it onto the surface, and what component acts as a pulling force down the plane? The ball weighs 600 N and the slope is 40° to the horizontal. (460 N, 386 N)



As can be seen, the pull and press are perpendicular components, and can be solved using the same methods as in previous examples.

Our superior knowledge of geometry allows us to see that by similar triangles, $\theta = 40^\circ$; from this, the problem can be solved:

$$\rightarrow \text{press, } F_{\perp} = F_g \cos 40 = 460 \text{ N}$$

$$\rightarrow \text{pull, } F_{\parallel} = F_g \sin 40 = 386 \text{ N}$$

As will be seen later, this way of creating components will be very useful in solving force-related problems for objects on a slope.

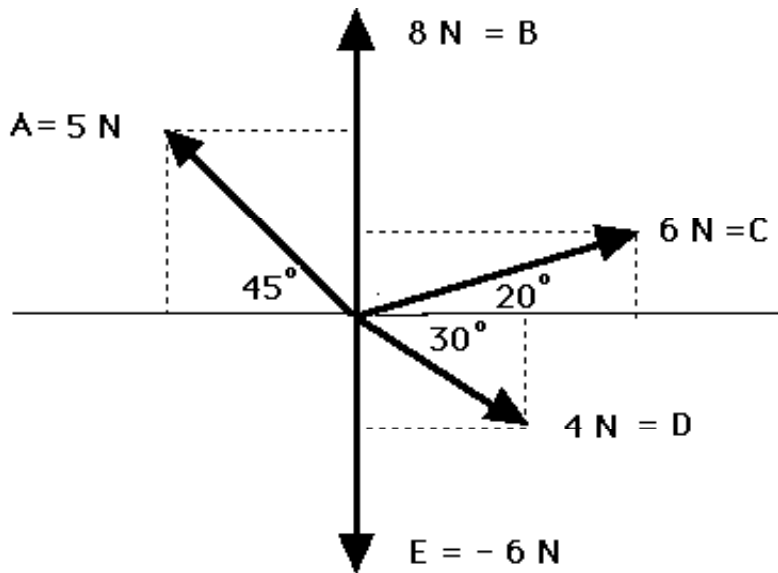
Finally, components can be used to add vectors when a large number of vectors (at least 4) are presented. Some or all of these steps can be used:

1. Draw all vectors on a set of X and Y axes
2. Draw in the X and Y components on the axes
3. Make a table adding the X and the Y components separately
4. Use vector addition to get the resultant of the X component sum and the Y component sum.

Now look at the following problem:

Add $8\text{ N (N)} + 5\text{ N (}45^\circ\text{N of W)} + 4\text{ N (}30^\circ\text{ S of E)} + 6\text{ N (}20^\circ\text{ N of E)} + 6\text{ N (S)}$

- Start with the drawing:



- Next, make a table of components and add them up; in this case, positive x-values are to the right, while positive y-values are 'up'

sum of X components

$$A_x = -5 \cos 45^\circ$$

$$B_x = 0$$

$$C_x = 6 \cos 20^\circ$$

$$D_x = 4 \cos 30^\circ$$

$$E_x = 0$$

+ 5.57 N (to the right)

sum of Y components

$$A_y = 5 \sin 45^\circ$$

$$B_y = 8$$

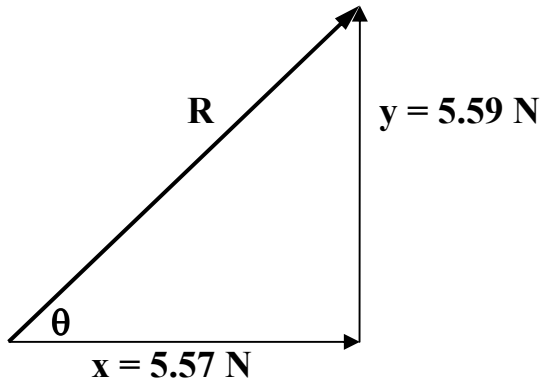
$$C_y = 6 \sin 20^\circ$$

$$D_y = -4 \sin 30^\circ$$

$$E_y = -6$$

+ 5.59 N ('up')

- Now find the magnitude of the resultant of these two values:



$$\rightarrow R = \sqrt{5.59^2 + 5.57^2} = 7.89 \text{ N}$$

- Finally, determine the direction:

$$\theta = \tan^{-1} \frac{5.59}{5.57} = 44.9^\circ$$

- The answer: **$R = 7.89 \text{ N}$ at 45° N of E**